where V is the velocity of the body in its orbit, k the constant of the system, ψ the angle between the tangent to the new orbit and the prolongation of the corresponding radius vector (r), r_0 the radius vector for the vertex of the old orbit, and p, a, and e are the semi-parameter, semi-major axis, and eccentricity of the new orbit. For the given point we have, as may easily be seen,

$$r = 2m,$$
$$\psi = 30^\circ,$$
$$r^2 \sin^2 \psi = m^2;$$

whence, by substitution in (a), we see that the new orbit will be an hyperbola, in which $p = 2m$, $a = -m$, $e = 1\ 3$. [*Ormond Stone.*]

EXERCISES.

26

DETERMINE the maximum right cone B inscribed in a given right cone A, the vertex of B being at the centre of the base of A. [*O. Root, Jr.*]

27

THE NUMBER of points common to three surfaces of the mth, nth, and pth degrees being in general mnp, find the co-ordinates of all the real and imaginary points of intersection of the three surfaces, $y^2 + x^2 + z^2 = 4x$, $x^2 = yz$, and $y^2 = x^4$. [*H. A. Newton.*]

28

GIVEN the perpendicular, median, and bisector issuing from one and the same vertex of a plane triangle and terminating in the opposite side, to construct the triangle and determine a formula for its area. [*Marcus Baker.*]

29

GIVEN a pair of points A,B, if C,D are such that $OD \cdot OC = OA^2 = OB^2$ and $AOC = AOD$, prove that the pair A,B bears similar relation to the pair C,D. Show, also, the existence of a pair E,F which bears the same relation to each of the pairs A,B and C,D. [*Wm. Woolsey Johnson.*]

30

INVESTIGATE formulæ for the logarithms of 13, 19, and 73 in terms of the logarithms of primes less than 13 and of $(N + 1/N)$ where $N = 132495$; 262143; 274625. [*F. H. Loud.*]

31

IN THE TRIANGLE ABC draw AD to the point D in BC; then will

$$AB^2 \cdot DC + AC^2 \cdot BD = BC \cdot AD^2 + BD \cdot DC \cdot BC.$$

[*J. R. Spiegel.*]

32

A TRIANGLE PQR is inscribed in triangle ABC. Determine the ratios in which P, Q, R divide the sides BC, CA, AB in order that AQR, BRP, CPQ may be respectively $\frac{1}{3}$, $\frac{1}{5}$, $\frac{1}{7}$, of ABC. [*W. M. Thornton.*]

SELECTED.—ALGEBRAIC EQUATIONS.

33

THE CUBIC $ax^3 + 3bx^2 + 3cx + d = 0$ has one or three real roots as its discriminant

$$D = (ad - bc)^2 - 4(b^2 - ac)(c^2 - bd)$$

is positive or negative. If D is zero, two roots are in general equal each to

$$\sqrt{\frac{c^2 - bd}{b^2 - ac}}.$$

34

SOLVE by quadratics the equations below and interpret their roots geometrically: —

$$x^4 - x^3\sqrt{15} + 4x^2 - 1 = 0,$$
$$x^4 - x^3\sqrt{3} - 2x^2 + 2x\sqrt{3} - 1 = 0.$$

35

SOLVE by quadratics the quartic

$$x^4 + ax^3 + bx^2 + ad^{\frac{1}{2}}x + d = 0.$$

36

SOLVE the cubic

$$\sqrt[3]{x} + \sqrt[3]{x - 7} = \sqrt[3]{3x + 3},$$

by eliminating its commensurable root.

37

If a, b, c, d, e be the roots of the quintic

$$x^5 + px^3 + qx^2 + rx + s = 0,$$

find the values of

$$a^2(de + eb + bc) + b^2(ea + ac + cd) + c^2(ab + bd + de) + d^2(bc + ce + ea) + e^2(cd + da + ab).$$

38

Show that among any three roots a, b, c of a biquadratic, as $x^4 + px^2 + q = 0$, this relations holds:—

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} = \frac{1}{a + b + c}.$$

CONICS.

39

On a chord MN of an ellipse as diameter a circle is drawn which cuts the curve in P and Q. Show that the ratio of the intercepts of MN and PQ on the axis major is $e^2 : 2 - e^2$.

40

The vertex of an hyperbola and one asymptote is fixed. Find the locus of the focus.

41

Find the equations to the conics which cut the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

orthogonally at all their common points.

42

From two points on Ox, equidistant from O, tangents are drawn to the conic

$$ax^2 + 2hxy + by^2 = 2x.$$

Find the locus of their points of concourse.

43

The polars of points on one of two equal circles relative to the other envelop a parabola.

44

Find the locus of the points from which tangents drawn to $Ax^2 + By^2 = 1$ are parallel to conjugates of $ax^2 + 2hxy + by^2 = 1$.